Multi-phase freak waves.

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Abstract:

There are many reasons to study multi-phase solutions of the nonlinear Schrödinger equation (NLS)

$$i\psi_z + \psi_{xx} + 2|\psi|^2\psi = 0. \tag{1}$$

- One of the possible origins of the freak waves is a synchronous addition of multi-phase waves maximums.
- Islas and Schober found in numerical simulations of the NLS equation that rogue waves develop for JONSWAP initial data are "near" 6-phase solutions [1].
- Rational solutions can be obtained using the degeneration of multi-phase solutions.
- Taking all the periods large enough we can make the difference between rational solutions and multi-phase solutions inside of parallelepiped of the periods arbitrarily small.

The results obtained in [2]–[3] for two-phase finite-gap solutions of NLS equation show that these solutions have a relatively simple behavior. In particular, the amplitude of two-phase solution for NLS equation is a doubly-periodic function both with respect to x and z variables:

$$|\psi|(x + X_k, z + Z_k) \equiv |\psi|(x, z), \quad k = 1, 2, \tag{2}$$

i.e. the two-phase solutions have the peaks on the nodes of the parallelogram of the periods. For elliptic two-phase solutions the number of peaks on the parallelogram of periods equals n, where n is a degree of mapping of the spectral curve to the torus. Three-phase finite-gap solutions have a more complicated behavior. They are periodic in 3D-space

$$|\psi|(x + X_k, z + Z_k, t + T_k) \equiv |\psi|(x, z, t), \quad k = 1, 2, 3, \tag{3}$$

where t is a modular parameter. During the talk we'll also describe the behavior of finite-gap solutions with higher number of phases in connection with freak waves.

References:

1. A. L. Islas and C. M. Schober, Phys. Fluids 17, 031701 (2005).
2. A. O. Smirnov, Theor. Math. Phys. 173(1), 1403-1416 (2012).
3. A. O. Smirnov, Math. Notes. (2013).